International Conference on Automation, Control, Engineering and Computer Science (ACECS'14)
Proceedings - Copyright IPCO-2014, pp.5-11
ISSN 2356-5608

Analysis of the Chaotic Discrete Model of Boost Converter

MeriemZhioua*[1], SafyaBelghith*[2]

** Electrical Department*
National Engineering School of Tunis (ENIT), Tunisia
[1]maryoumazhioua@gmail.com
[2]safyabelghith@yahoo.fr

***Abstract*—The hybrid dynamical systems (SDH) are complex systems that involve both continuous-time and discrete-time dynamics. This hybrid aspect presents a variety of nonlinear phenomena such as bifurcations and chaos, which is the case of switching power circuits. As the discrete time model is more accurate and simpler, in this paper, our aim is to analyze the chaotic behavior of the current-programmed boost converter as a case of study from an exact discrete time model. The MATLAB simulation results are then compared to the ones using the continuous time model and validated from the responses of the system simulated in PSIM. By considering the reference currentas a parameter bifurcation, we present the strange attractor and the bifurcation diagram. We also calculate its Lyapunov exponents to determinate if the DC-to-DC boost converter behaves in a chaotic manner.**

***Keywords*— dynamical hybrid system, DC-DC boost converter, current programmed, exact discrete time model, bifurcation, Lyapunov exponents.**

I. INTRODUCTION

Lately, research on hybrid systems has known a strong interest, motivated primarily by the industrial applications. Indeed, industrial process control requires the control of the interactions in the time between discrete events generated by the computer and the continuous dynamics of the physical phenomenon. On the other hand, the hybrid nature of the system may come from the system itself, which is for example the case of the switching power converter circuits.

DC-DC converters (Buck, Boost, Buck-Boost…) are a typical example of power electronic circuits and are characterized by cyclic switching of circuit topologies [8]. Despite their simple schemes, the switching aspect of such systems makes it complex and may have a variety of nonlinear behaviorsincluding bifurcations, quasi-periodicity and chaos[2],[5],[9]. In academic community and industry, many research teams work on analysis and control of these systems in order to provide efficient control laws that are stable within the operating range, in response to certain specifications and taking into account the problem of being industrially applicable.

The hybrid aspect of power circuits makes it difficult to analyze and predict its behavior. Thus, as it is known, exact discrete-time model has been successfully used to study the nonlinear dynamics of switching power converters [3], [12].

A simple example of DC-DC power converter is theDC-to-DC boost converter. Considering its hybrid aspect, this chapter aims atexaminingand studying the dynamic behavior of the current-mode controlled boost converter. We show also withMatlab simulations that variation of the current reference can lead to interesting route to chaos.The simulation results are approved by calculating Lyapunov exponents.

The paper is organized as follows. Section 2 provides an overview of bifurcation, chaotic theory and Lyapunov exponents. The operating mode of boost converter is defined in section 3 using PSIM simulation. We also present the state space model and the exact-discrete time model of the circuit in section 3.PSIM and MATLAB simulation resultsof discrete and continuous time models are included in section 4. We show also the analyze of the chaotic behavior of the converter by presenting the bifurcation diagram and calculating the Lyapunov exponents.Section 5 presents the conclusion of this work and shows briefly our future researches.

II. BASIC PRINCIPLES OF BIFURCATION, CHAOTIC THEORY AND LYAPUNOV EXPONENTS

The study of nonlinear dynamics of DC-DC converters started in 1984 by Brockett's and Wood's research[4]. Since then, chaos and nonlinear phenomena in power electronic circuits have stolen the spotlight and have attracted the attention of different research groups[4].

"Chaos theory is the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems." It studies the behavior of dynamical systems that are hypersensitive to initial condition [7].

Generation of a chaotic system is not immediate. Indeed, the system does not change of nonexistent state to a chaotic state without going through transitions.

Consider the nth-order discrete dynamical system parameterized by a single parameterμ:

$$x_{n+1} = f(x_n, \mu) \tag{1}$$

We assume that $f$is a smooth function.

By changing the system parameterμ, the limit sets of the system also change [1], [3]. Typically, a small change in μ produces small quantitative changes in a limit set. For instance, perturbing μ could change the position of a limit set slightly, and if the limit set is not an equilibrium point, its shape or size could also change. There is also the possibility that a small change in μ can cause a limit set to undergo a qualitative change[11], [13]. Such a qualitative change is called a bifurcation and the value of μ at which a bifurcation occurs is called a bifurcation value [6].

ISSN 2356-5608

Doubling-period bifurcation is decidedly the most known one.It corresponds to the instability of a stable period-k solution of a map and the generation of a stable period-2k solution. A sequence of period doubling bifurcations where the system proceeds fromperiod-one to period-two, then from period-two to period-four, etc… leads to chaos[8].Indeed, the bifurcation values accumulate at a particular parameter value for which the system becomes chaotic [11]. The most commonly used tool for analysis of the systems dynamics when the system has chaotic behavior is the bifurcation diagram.

The chaotic regime of the converters have been detected and analyzed through Lyapunov exponents [10]. As it is well known, Lyapunov exponents measure the exponential rates of divergence or convergence of nearby orbits of an attractor in the state space.A positive Lyapunov exponent found indicates that the system is chaotic. In fact, in chaotic regions the exponent is positive because it indicates the rate at which one loses the ability to predict the system response. Considering the discrete dynamical system:

$$x_{n+1} = f(x_n) \qquad (2)$$

whose the solution is defined in terms of the initial condition x_0:

$$x_n(x_0) = f(x_{n-1})f(x_{n-2}) \dots . f(x_1)f(x_0) \qquad (3)$$

Lets the variational system associated:

$$\delta x_{n+1} = Df(x_n)\delta x_n \qquad (4)$$

where$Df(x_n)$ is the Jacobian of $f(x_n)$.

Thereby, the Lyapunovexponentsare calculated as follows:

$$\lambda(e^n, x_0) = \lim_{n\to\infty} \log \frac{\left\| \mathrm{Df}_{x_0}\mathrm{e}_1 \wedge \dots \wedge \mathrm{Df}_{x_0}\mathrm{e}_n \right\|}{\| \mathrm{e}_1 \wedge \dots \wedge \mathrm{e}_n \|} \qquad (5)$$

Where $e^n$is a subspace of dimension n of the tangent space E_{x_0} , and { $\mathrm{e_i}$ }, $i = 1,2, \dots, n$ the base of e^n , $\wedge$ represents the exterior product and $\|\dots\|$ represents the norm.

III. CURRENT-MODE CONTROLLED BOOST CONVERTER IN OPEN LOOP

A. System description

PSIM is a program designed for power electronics simulation.

By default, components are ideal, which allow us to focus on understanding the functioning of the circuit.

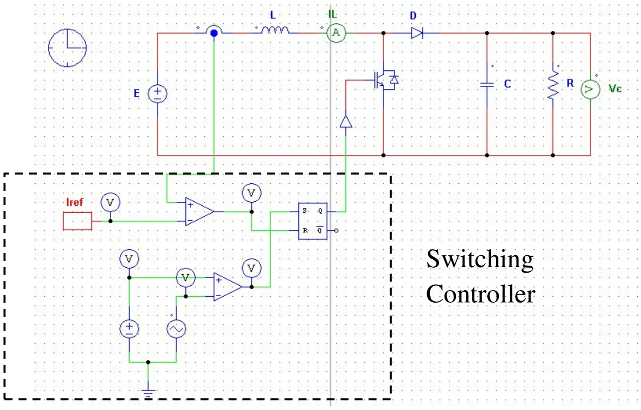


Fig. 1: PSIM simulation model of boost converterwith current control mode

A boost converter is a DC-DC power converter with an output voltage greater than its input voltage.It consists of two parts: a converter and a switching controller, as it is shown in Fig. 1.

The circuit includes the following elements: an inductor, a capacitor, a diode, a controlled electronic switch and a load resistance connected in parallel with the capacitor.

The working principle of the boost converter can be explained as follows. The switching controller drives the general circuit. It compares the inductor current$\mathrm{I_L}$with the reference current $\mathrm{I_{ref}}$ and generates the on/offcontrolling signal for the switch by a set-reset Flip-Flop. Indeed,during the switch on period (the switch is closed), the diode is blocked and the inductor current I_L increases until attainingthe reference current I_{ref} which leads to make the set-reset flip-flop turning the switch off until reaching the next pulse, in which the switch is closed again.

Assuming that we operate in continuous conduction mode (CCM), the boost converter switches between two states: switch on and switch off. The general equation that governs the operation of the boost converter is:

$$\dot{x}(t) = A(q)x(t) + B(q) with\ q \in \{1,2\}. \qquad (6)$$

For $q = 1\ and q = 2$, we obtain:

$$S_{on}: \quad \dot{x} = A_1 x + B_1 \qquad (7)$$

$$S_{off}: \quad \dot{x} = A_2 x + B_2 \qquad (8)$$

Where $x = \begin{bmatrix} Vc \\ iL \end{bmatrix}$ is the state vector, and the matricesA_1,A_2,B_1, B_2 are given by:

$$A_1 = \begin{bmatrix} \frac{-1}{RC} & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{E}{L} \end{bmatrix}$$

ISSN 2356-5608

$$A_2 = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{E}{L} \end{bmatrix}$$

B. *Continuous time modeling for a boost power converter*

The continuous time modeling of boost converter is presenting using Simulink-Stateflowas it is shown in Fig. 2.

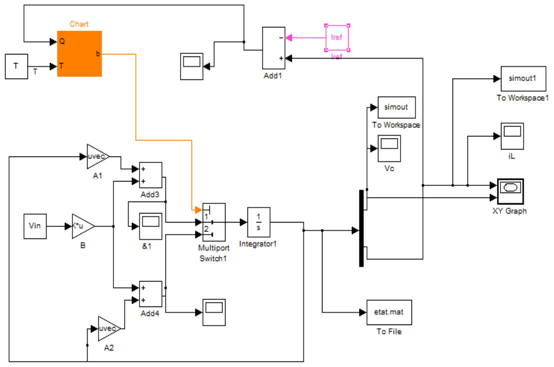


Fig. 2: Simulink-Stateflow simulation model of boost converter with current control mode.

The Stateflow block Simulink is a graphical tool for design and development used in the modeling of state machines.Using this tool, we were able to simulate the behavior of the continuous model of the Boost converter.

C. *Exact discrete-time modeling fora boost power converter*

The aim of this part is to develop an iterative application that gives the ability to switch of a continuous complex dynamic system to a simplified discrete dynamic. This modeling will be able to study the behavior of the system via MATLAB simulation.

In order to model the dynamic of a two-state switching converter operated in the continuous conduction mode, it should be from its continuous model described as:

$$\begin{cases} \frac{dx}{dt} = A_1 x + B_1 e(t) pour nT \leq t < nT + dT \\ \frac{dx}{dt} = A_2 x + B_2 e(t) pour nT + dT \leq t < (n+1)T \end{cases} \quad (9)$$

wherex is the state vector, e is the input voltage and d is the duty ratio of the switch.

The exact discrete-time model can then be expressed as:

$$x\big((n+1)T\big) = e^{A_2(1-d)T}\left[e^{A_1(dT)}x(nT) + dTB_1 e\right] + \left(e^{A_2(1-d)T} - I\right){A_2}^{-1}B_2 e \quad (10)$$

$$d = \left(I_{ref} - I_L(nT) \right) \frac{L}{Te} \quad (11)$$

where I_{ref} is the reference current,I_L is the inductor current, L is the independence, and e is the input voltage.

IV. SIMULATION RESULTS

Responses of exact-discrete time model and continuous time model were simulated in MATLAB for step changes in the reference current from 0.7 A to 1.5 A . The circuit parameters are set as follows: $L = 1.5$ mH , $C = 5$ μF , $R = 40$ Ω, $T = 100$ μs, $V_{in} = 10$ V.

A. *PSIM simulation results*

Fig. 3 shows the functioning of boost converter at periodic state ($I_{ref} = 0.7\ A$) using PSIM.

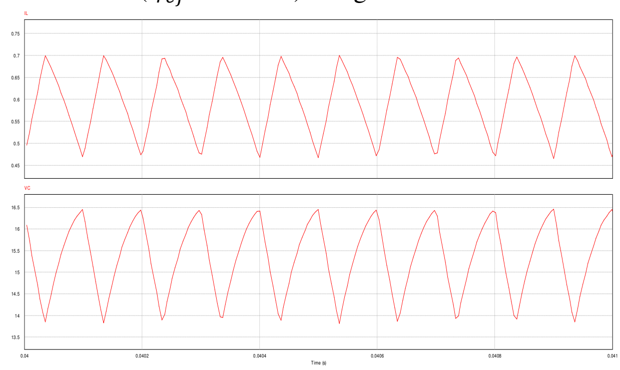


Fig. 3: Simulated inductor current and capacitance voltage of boost converter using PSIM at periodic state ($\mathbf{I_{ref} = 0.7\ A}$)

By increasing I_{ref} to 1.5A, the system goes into a chaotic state as it is shown in fig. 4:

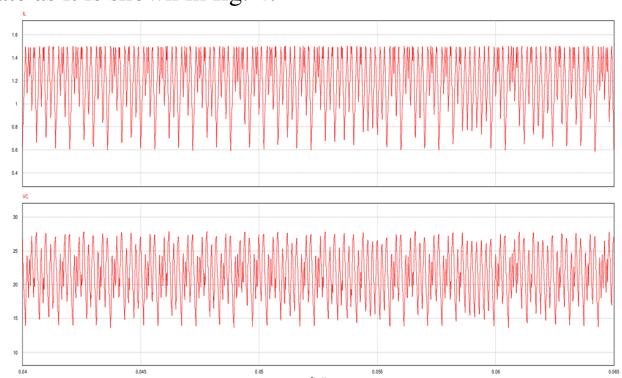
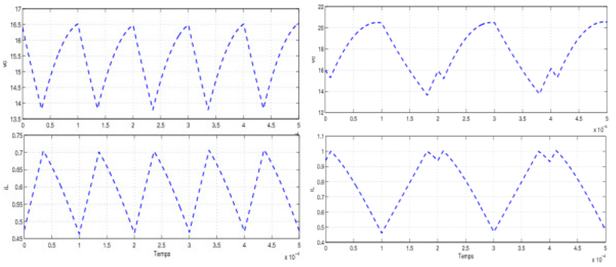


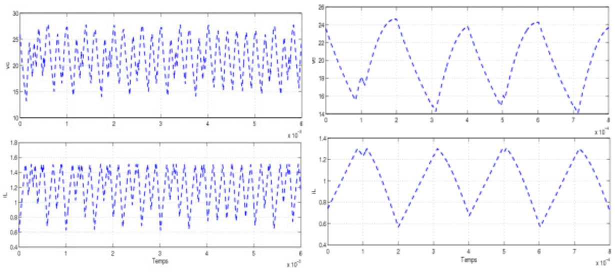
Fig. 4: Simulated inductor current and capacitance voltage of boost converter using PSIM at chaotic state ($\mathbf{I_{ref} = 1.5A}$)

ISSN 2356-5608

B. Matlab simulation results: Continuous time model

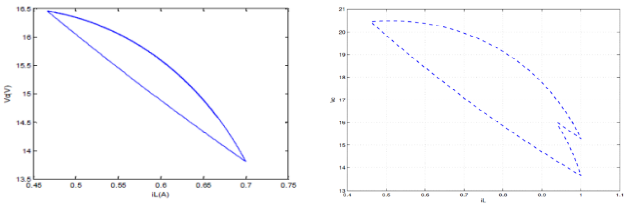


(a) Period-1 $(\mathbf{I_{ref}} = \mathbf{0.7A})$ (b) Period-2 $(\mathbf{I_{ref}} = \mathbf{1A})$

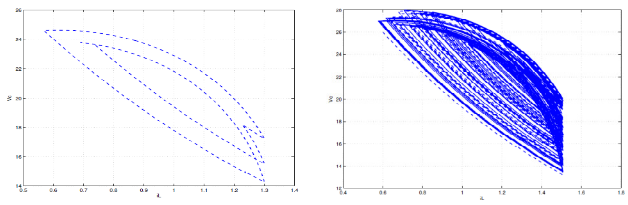


(c) Period-4 $(\mathbf{I_{ref}} = \mathbf{1.3A})$ (d) Chaotic state$(\mathbf{I_{ref}} = \mathbf{1.5A})$

Fig. 5: Simulated capacitance voltage and inductor current of continuous time model of boost converter in response to a step change in the reference current from $\mathbf{0.7}$ **A to** $\mathbf{1.5}$ **A**



(a) Period-1 $(\mathbf{I_{ref}} = \mathbf{0.7A})$ (b) Period-2 $(\mathbf{I_{ref}} = \mathbf{1A})$

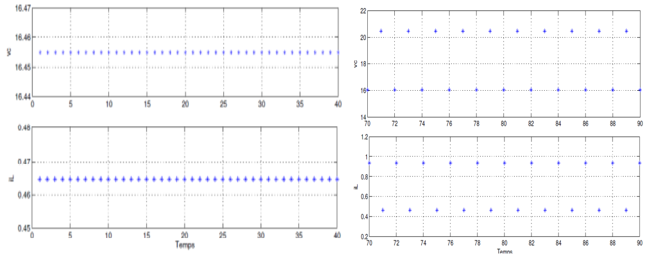


(c) Period-4 $(\mathbf{I_{ref}} = \mathbf{1.3A})$ (d) Strange attractor$(\mathbf{I_{ref}} = \mathbf{1.5A})$

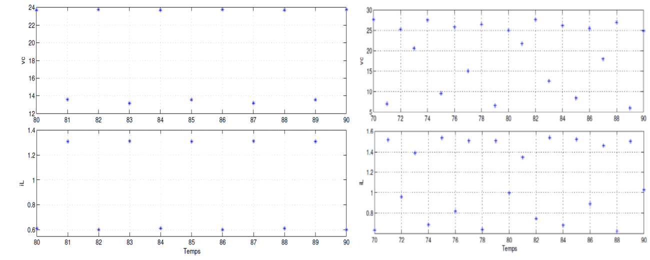
Fig. 6: Simulated phase plane $\mathbf{V_c}$-$\mathbf{i_L}$ ofcontinuous time model of boost converter in response to a step change in the reference current from $\mathbf{0.7}$ **A to** $\mathbf{1.5}$ **A**

Concerning the continuous time model, we note in Figures5 and 6 that for $\mathbf{I_{ref}} = \mathbf{0.7}$ **A**, we are in a periodic regime. By increasing $\mathbf{I_{ref}}$ over $\mathbf{0.7}$ **A**, we will get period-2 for $\mathbf{I_{ref}} = \mathbf{1A}$, then period-4 for $\mathbf{I_{ref}} = \mathbf{1.3}$ **A**, and thereafter the chaos starts from $\mathbf{I_{ref}} = \mathbf{1.5}$ **A.**

C. Matlab simulation results: Exact-discrete time model

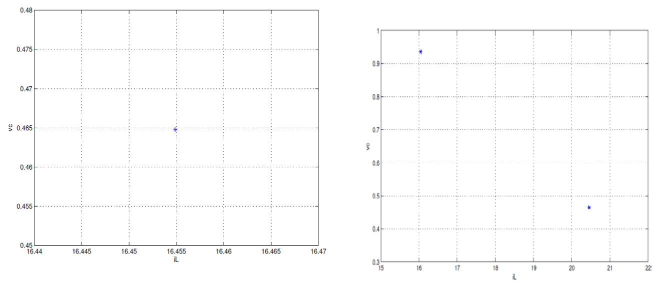


(a) Period-1 $(\mathbf{I_{ref}} = \mathbf{0.7A})$ (b) Period-2 $(\mathbf{I_{ref}} = \mathbf{1A})$

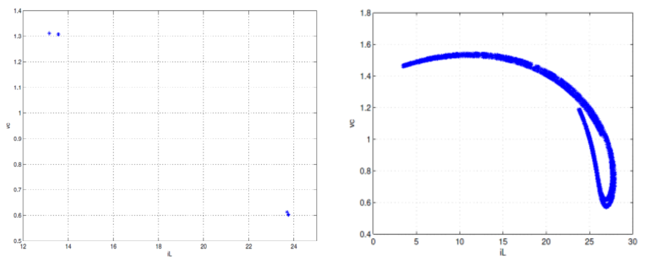


(c) Period-4 $(\mathbf{I_{ref}} = \mathbf{1.3A})$ (d) Chaotic state$(\mathbf{I_{ref}} = \mathbf{1.5A})$

Fig. 7: Simulated capacitance voltage and inductor current of exact-discrete time model of boost converter in response to a step change in the reference current from $\mathbf{0.7Ato1.5A}$



(a) Period-1 $(\mathbf{I_{ref}} = \mathbf{0.7A})$ (b) Period-2 $(\mathbf{I_{ref}} = \mathbf{1A})$



(c) Period-4 $(\mathbf{I_{ref}} = \mathbf{1.3A})$ (d) Strange attractor$(\mathbf{I_{ref}} = \mathbf{1.5A})$

Fig. 8: Simulated phase portrait V_c-i_L of exact-discrete time model of boost converter in response to a step change in the reference current from 0.7 A to 1.5 A

Figures 7 and 8 show period-doubling cascade from period 1 to chaos.The first bifurcation takes place at $I_{ref} = 1$ A where the period 1 bifurcates to period 2. The period 2 behavior again bifurcates to period 4 behavior at $I_{ref} = 1.3$ A.

D. Bifurcation structure

Figures 9 and 10 show the bifurcation diagram and its Lyapunov exponents.

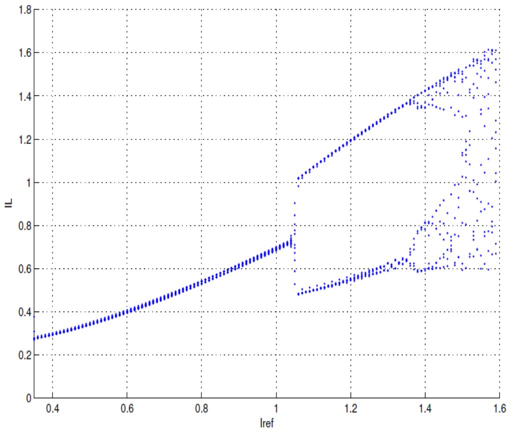
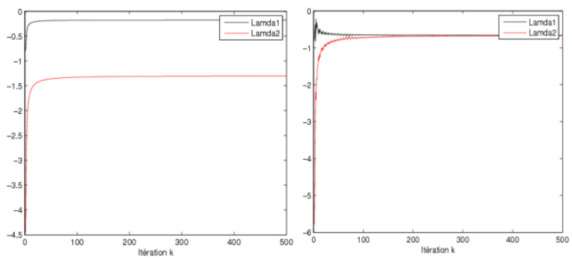


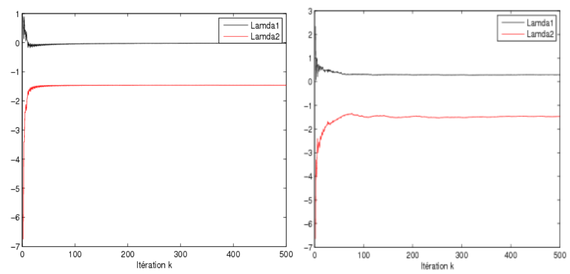
Fig. 9: Bifurcation diagram

Fig.9 shows an important sequence of bifurcations, where the system proceeds from period one to period two, then from period two to period four, and eventually the onset of chaotic regime is observed. This kind of bifurcation is called flip bifurcation or period doubling bifurcation.

Fig.10shows the convergence speed of Lyapunov exponents.



(a) Period-1 ($I_{ref} = 0.7$ A) (b)Period-2 ($I_{ref} = 1$ A)



(c) Period-4 ($\mathbf{I_{ref} = 1.3A}$) (d) Chaotic state($\mathbf{I_{ref} = 1.5A}$)

Fig. 10: Lyapunov Exponents

It is noted in figure 10-(d) that the Lyapunov exponents when $I_{ref} = 1.5$ A, are:
$\lambda_1 = 0.291 > 0$, $\lambda_2 = -1.468$.This may confirm that the system has a chaotic behavior.

V. CONCLUSION

The exact discrete time model provides more accurate presentation of hybrid dynamical systems. Therefore, it has been used in this work to study the chaotic behavior of the current-mode control boost converter. Variation of the reference current as a bifurcation parameter shows that the circuit tracks a period doubling until chaos. This is illustrated in the bifurcation diagram. On the other side, calculating theLyapunov exponents provides information about the circuit's behavior. Indeed, the positive exponent found, proves the chaotic behavior of the boost converter.

In this work, the DC-to-DC boost converter was chosen as a case study. For future, our objective is to generalize this study for any kind of converter, and to apply control laws for the control of chaos in power converters.

REFERENCES

[1] Aldahim, G., Oirekozek, H., &Sakka, Z. (2013). Impact of Inductor Resistance on the Dynamic Behaviour of a DC–DC Boost Converter Using Bifurcation and Chaos Theory. *Energy Procedia*, 36, 1025-1034.

[2] Chen, M. L., & Ma, W. M. (2006). Bifurcation and chaos in multi-parallel-connected current-mode controlled boost DC-DC converters. *Frontiers of Electrical and Electronic Engineering in China*, 1(2), 183-187.

[3] Di Bernardo, M., &Vasca, F. (2000).Discrete-time maps for the analysis of bifurcations and chaos in DC/DC converters. Circuits and Systems I: Fundamental Theory and Applications, *IEEE Transactions on*, 47(2), 130-143.

[4] El Guezar, F., Bouzahir, H., Acco, P., Afdel, K., & Fournier-Prunaret, D. (2007, March). Modeling and Simulation in Scicos: A Case Study. In *Computational Intelligence and Intelligent Informatics, 2007. ISCIII'07.International Symposium on* (pp. 105-110). IEEE.

[5] Kavitha, A., & Uma, G. (2010). Control of chaos in SEPIC DC-DC converter.*International Journal of Control, Automation and Systems*, 8(6), 1320-1329.

[6] Larson, L. E., Tsimring, L. S., & Liu, J. M. (2006). *Digital communications using chaos and nonlinear dynamics*. New York: Springer.

[7] Li, H., Zhang, B., Li, Z., Halang, W. A., & Chen, G. (2009). Controlling DC–DC converters by chaos-based pulse width modulation to reduce EMI. *Chaos, Solitons& Fractals*, 42(3), 1378-1387.

[8] Matsuo, A., Asahara, H., &Kousaka, T. (2012). Bifurcation structure of chaotic attractor in switched dynamical systems with spike noise. *Chaos, Solitons& Fractals*, 45(6), 795-804.

[9] Morel, C. M. (2005). *Analyse et contrôle de dynamiques chaotiques, application à des circuitsélectroniques non-linéaires* (Doctoral dissertation, Angers).

[10] Müller, P. C. (1995). Calculation of Lyapunov exponents for dynamic systems with discontinuities. *Chaos, Solitons& Fractals*, 5(9), 1671-1681.

[11] Parker, T. S., & Chua, L. O. (2002). Practical numerical algorithms for chaotic systems, 1989. *Springer, New York, Falsaperla P., Fonte G., Salesi G., Found.Phys*, 32, 267.

[12] Rajasekaran, V., Sun, J., & Heck, B. S. (2003). Bilinear discrete-time modeling for enhanced stability prediction and digital control design. Power Electronics, *IEEE Transactions on*, 18(1), 381-389.
[13] Zhou, X., & Li, J. (2012). Chaos Phenomena in DC-DC Converter and Chaos Control. *Procedia Engineering*, 29, 470-473.